

# Subjective Well-being, Consumption Comparisons and Optimal Income Taxation.\*

Sean Slack<sup>†</sup>   David Ulph<sup>‡</sup>

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## Abstract

We introduce reference consumption into the standard utility function from optimal tax analysis. Individuals compare their consumption ‘narrowly’ with those of the same productivity, or ‘broadly’ with the average consumption across society. In both Narrow and Broad equilibria reference consumption is an increasing function of the tax parameters, so generating new theoretical results. Individual well-being decreases with the net wage (net-of-tax) rate for low productivity workers under Narrow (Broad) comparisons; thus adjusting redistributive taxation considerations. Further, in both cases reference consumption distorts labour supply away from the social optimum level; giving a distortion-correcting role for taxation.

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<sup>†</sup>School of Economics & Finance, University of St. Andrews, Castlecliffe, The Scores, St. Andrews, Fife, KY16 9AR, Scotland, U.K. Email: sean.ses34@gmail.com. The majority of this research was undertaken, following my doctoral studies, as a research assistant at the University of St. Andrews (funded by the AXA research fund).

<sup>‡</sup>School of Economics & Finance, University of St. Andrews, Castlecliffe, The Scores, St. Andrews, Fife, KY16 9AR, Scotland, U.K. Email: du1@st-andrews.ac.uk

# 1 Introduction

The standard optimal income tax framework (i.e., [Mirrlees, 1971](#); [Sheshinski, 1972](#)) assumes that individual well-being is independent of the outcomes of others. This is in contrast with the substantial body of empirical work suggesting that income and consumption comparisons are a key-driver of one’s subjective well-being ([Blanchflower and Oswald, 2004](#); [Clark and Oswald, 1996](#); [Ferrer-i Carbonell, 2005](#); [Layard, 2005](#); [Layard et al., 2009](#); [Senik, 2009](#); [Clark et al., 2008](#)).<sup>1</sup> Much of this research is based on survey evidence in which the main socio-economic variable available to the researcher is income: it is an open question whether it is income or consumption comparisons that are relevant, but in the context of the framework in this paper the two coincide. With this in mind, our paper analyses the optimal income tax problem when individual preferences are defined very generally over own-consumption and labour time (as in the standard model) *and* the average consumption of some reference group.

Our question is as follows: how do the conventional results from the optimal linear income tax framework change when individual well-being is decreasing in the average consumption of some reference group and, further, the level of reference group consumption may be an argument of individual choices, thus generating *Keeping up with the Joneses* behaviour? In answering this question, we are also responding to recommendations that public economic theory should incorporate key insights from the extensive literature on subjective well-being ([Layard, 2006](#); [O’Donnell et al., 2014](#)).

An empirical literature helps inform the appropriate choice of theoretical framework, in particular providing evidence on the composition of reference groups; the size of income/consumption externalities; and the behavioural implications of income comparisons. First, [Clark and Senik \(2010\)](#) elicit from the European Social Survey (Wave 3) that a majority of individuals compare their income with others and, further, of those who compare, colleagues are the most prevalently reported reference group. Second, a number of studies suggest that an increase in reference group income or corresponding reduction in own-income reduce well-being by similar magnitudes ([Luttmer, 2005](#); [Ferrer-i Carbonell, 2005](#)). Third, there is evidence that income comparisons have an upwards effect on labour supply: an individual whose income falls behind that of the reference group may increase their work hours in a bid to ‘*Keep up with the Joneses*’ ([Pérez-Asenjo, 2011](#)). This is also consistent with the prevailing view that individuals do not compare their leisure with that of others ([Layard, 2006](#)).<sup>2</sup>

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<sup>1</sup>This literature has largely spawned from the well-documented ‘Easterlin Paradox’: substantial increases in real income per capita across developed countries have not been accompanied by marked increases in average reported well-being; though within-country higher income individuals tend to be happier than lower income individuals ([Easterlin, 1974, 1995, 2001](#)). Relative income theory ([Duesenberry, 1949](#)) provides a possible resolution: if individual preferences are defined over consumption *relative* to the unweighted average in society, a *ceteris paribus* increase in a given individual’s income will increase their well-being, but an increase in the average national income will generate no such increase in individual well-being (see also [van de Stadt et al., 1985](#)).

<sup>2</sup>Though since [Veblen \(1899\)](#) it has been recognised that leisure may play a role in ‘displaying’ relative con-

There is also experimental evidence that individuals may trade-off income for higher relative standing in choosing their preferred society: this is illustrated, for example, in [Johansson-Stenman et al. \(2002\)](#), where individuals compare societies of equal income inequality but different relative positions for their hypothetical grandchildren (also see [Alpizar et al., 2005](#); [Fehr and Schmidt, 1999](#)).

In our framework individuals may compare their consumption ‘narrowly’ with those of the same productivity, or ‘broadly’ with the average consumption across society as a whole.<sup>3</sup> Individuals choose their Marshallian labour supply taking as given the reference consumption level; but in both the Narrow and Broad cases the equilibrium level of reference consumption will be a function of the tax parameters (and in the Narrow case also a function of productivity). Consequently, a number of new theoretical results arise relative to the conventional theory.

Our first result concerns well-being and the net wage/net-of-tax rate. In the conventional theory an increase in the net wage (net-of-tax) rate increases the well-being of workers at a rate proportional to labour supply (gross earnings). In our framework, there is also a second opposing effect. Under Narrow comparisons an increase in the net wage rate increases average peer consumption, which acts to lower well-being because reference consumption is a negative externality. For workers with productivity close to the reservation productivity - and so very low labour supply - this second effect dominates the conventional effect and well-being falls with the net wage rate. The implication is that, in contrast to the conventional theory, the worst-off in society are no longer the voluntarily unemployed or those of lowest productivity, but instead workers of low productivity. Next, under Broad comparisons average consumption is independent of productivity and so well-being is unambiguously increasing in individual productivity<sup>4</sup>. However, an increase in the net-of-tax rate is well-being reducing for workers of low productivity. The intuition is analogous to the Narrow comparison case: the negative externality of increased reference consumption offsets the traditional effect. In the Broad comparison case we must thus distinguish between changes in individual productivity and the net-of-tax rate, whereas in the Narrow case both arguments are summarised via the net wage rate.<sup>5</sup> Finally, these well-being results hold for both the cases where labour supply is independent of reference consumption (the pure negative externality case) and where labour supply is an increasing function of the reference consumption level.

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sumption disparities: individual consumption is time consuming and so higher consumption may require more leisure, whilst individuals taking leisure may notice more the consumption of others (see [Aronsson and Johansson-Stenman, 2013](#)).

<sup>3</sup>Our framework is static and so income is equivalent to consumption.

<sup>4</sup>We think of this as a cross-sectional result.

<sup>5</sup>The result that an *individual’s* well-being falls with the net-of-tax rate continues to hold in a ‘[Piketty and Saez \(2012\)](#)’ style framework where individuals differ in *both* their preferences and earnings ability. However, because such a framework does not permit interpersonal comparisons all we can say is that the sign of the inequality has changed relative to the standard theory. For this reason, a framework with homogenous preferences affords us more theoretical insight.

This first result has implications for the how the social marginal welfare weight (smww) changes with individual productivity. In modern tax analysis, the smww captures the value, in units of public funds, that society places on awarding a given individual an additional one unit of income ([Piketty and Saez, 2012](#)). In the conventional theory the smww is proportional solely to the marginal utility of consumption (and any distributional weights<sup>6</sup>): the effect of behavioural responses on well-being cancel by the Envelope theorem. In our framework, an increase in unearned income also increases average reference consumption, and so the smww differs from the standard theory. A crucial consideration for redistributive taxation is how the smww differs across individuals, and so here across productivity levels. The non-monotonicity of well-being with the net-wage rate under Narrow comparisons suggests that the smww may fall less rapidly with productivity under Narrow comparisons relative to both the Broad comparison case and the standard theory.

It is widely recognised that when individuals have concerns about relative consumption there is an externality at work that can lead equilibrium labour supply to be ‘distorted’ away from the socially optimum level. Defining the social optimum is in general a non-trivial exercise. The Narrow case is the most straightforward: individuals behave in a Nash fashion and so fail to recognise the interplay between their own decisions and the Nash consumption level when choosing labour supply. A social planner would recognise that in equilibrium own-consumption equates with reference consumption and so fully internalise the externality when choosing labour supply.<sup>7</sup> However, in the Broad case a given individual does not affect the average consumption level across society and so the concept of a social optimum is more nuanced. To capture the social optimum we use the notion of Kantian calculus: individuals choose their labour supply under the Kantian conjectural variation that if they increase their own income so too will everyone else, such that a change in own consumption generates a corresponding change in average consumption. The Kantian equilibrium concept is increasingly employed in the study of externalities (e.g., see [Roemer, 2010](#)) and here generates an expression for the social optimum that is analogous to the Narrow case.

The three results discussed above help inform our derivation and interpretation of the optimal linear tax rates under both Narrow and Broad references. In both cases the optimal tax rate is characterised implicitly by three considerations: distortion-correction; the covariance between the smww and relative gross earnings (the equity term in conventional tax expressions); and the aggregate earnings elasticity (the efficiency term in conventional tax expressions).

The distortion-correction terms captures the fact that reference consumption is an externality that individuals either do not account for (in the Narrow case) or do not influence (in the Broad

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<sup>6</sup>I.e., concave transformations of utility

<sup>7</sup>This distortion between Nash labour supply and the social optimum can in principle be corrected by a Pigovian tax on each net wage (the informational requirements are therefore strict). [Layard \(2006\)](#) and [Ljungqvist and Uhlig \(2000\)](#) derive this in the context of a homogenous population.

case). *Ceteris paribus*, the optimal tax rate rises to reduce the size of these externalities on average; or equivalently to partially correct the distortion between individual choices and the socially optimum choices described above.

The covariance between the smww and relative gross earnings corresponds to the equity term found in conventional tax expressions. In conventional tax theory the smww falls with productivity, whilst earnings rise. The covariance is therefore negative, thus prompting a desire to redistribute from those of high gross earnings (low smww) to those of low gross earnings (high smww). As discussed above, our well-being results suggest that traditional redistributive considerations may be somewhat lessened in the Narrow comparison case, relative to both the Broad comparison case and the standard theory.

As in the conventional framework our tax expression features the aggregate earnings elasticity in the denominator (Piketty and Saez, 2012). At the level of generality of the analysis, there is not too much we can say about this term. The presence of *Keeping up with the Joneses* (KUJ) multiplier effects renders labour supply more responsive to the tax parameters, and this will affect the aggregate elasticity relative to the standard theory. In reality of course, we can estimate aggregate elasticities and so it is unclear how much the theory adds in this regard.

Our numerical example seeks to shed light on the balance of these considerations. We augment the standard isoelastic functional form (Saez, 2001) to include reference consumption, and do so in a general way whereby reference consumption can be either a pure negative externality or both a negative externality and an argument of labour supply. The optimal tax rate is increasing in the weight that individuals place on reference consumption under both Narrow and Broad comparisons. However, the optimal tax rate is lower under Narrow comparisons than under Broad comparisons, as suggested by the implications of our well-being results for the smww. Finally, the optimal tax rate is lower when there are KUJ effects.

As will be discussed below, our paper is not the first to suggest that tax rates rise with relative consumption concerns. However, we see our main contribution as identifying new well-being results that add to our understanding of how tax considerations are affected by relative consumption: our well-being results demonstrate that redistributive considerations differ depending on reference group composition, whilst KUJ effects may heighten conventional efficiency concerns, both of which may temper the extent to which tax rates rise with relative consumption concerns.

We proceed to discuss the existing theoretical literature.

## 1.1 Related Literature

A number of papers examine the implications of consumption comparisons for optimal income taxation. Boskin and Sheshinski (1978) analyse optimal linear income taxation when individual utility/well-being is decreasing in the average consumption level across society. The disutility of individual effort is captured in consumption units and so optimal effort is independent of average

consumption, which thus acts as a pure negative externality (so in contrast to our framework the reference consumption level is not an argument of labour supply/effort).<sup>8</sup> Their finding that the optimal tax rate increases with the weight individuals place on consumption comparisons is highly intuitive: an increase in the tax rate (and accompanying increase in the lump-sum benefit) acts to lower average consumption, and so the externality. [Layard \(2006\)](#) analyses the case where consumption comparisons are a negative externality and may also be an argument of labour supply.<sup>9</sup> The author focuses purely on efficiency considerations (all individuals have the same productivity) and demonstrates that the socially optimum labour supply can be attained via a (Pigovian) tax on income. In contrast to the conventional theory, these behavioural responses (corrections) to taxation are not a deadweight loss (see also [Blomquist, 1993](#)).<sup>10</sup>

In a nonlinear income tax context, a few papers demonstrate that an increase in the degree of concern of relative consumption increases (i) marginal tax rates ([Oswald, 1983](#)); and further, (ii) the progressivity of the tax schedule ([Kanbur and Tuomala, 2013](#)). The latter analysis of Kanbur and Tuomala also suggests that an increase in pre-tax inequality lessens the above two effects. These papers take the reference consumption level to be the average in society. Alternatively, [Micheletto \(2010\)](#) explores nonlinear income taxation in a three-ability type model where individuals compare their consumption with that of the next highest ability type. To the extent that the shadow prices attached to self-selection constraints are high and the disutility of reference consumption is increasing in labour supply, a reduction in the marginal tax rates faced by higher types discourages mimicking behaviour and may be optimal.

In the static optimal tax framework income equals consumption, and so consumption comparisons are equivalent to income comparisons: the same is not true in a dynamic setting with savings. A growing literature explores the implications of consumption comparisons for optimal taxation within an overlapping generations (OLG) framework. [Aronsson and Johansson-Stenman \(2008\)](#) analyse optimal nonlinear labour and capital income taxation in a two-ability type, two-period model. Tax revenues are used for both redistribution and public good provision.<sup>11</sup> Individuals compare their periodic consumption with the average consumption across the economy; where the comparisons are captured through the difference between own and reference consumption. The optimal marginal labour income tax rates are increasing in the weight individuals place on relative consumption: this acts to lower average consumption. [Aronsson](#)

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<sup>8</sup>Formally, preferences in [Boskin and Sheshinski \(1978\)](#) are  $u(c, \bar{c}; \theta)$ ; where  $c$  is own-consumption,  $\bar{c}$  is average consumption and  $\theta$  is the weight placed on comparisons. If  $z$  is gross earnings,  $g(z)$  the disutility of earnings in consumption units, and  $\alpha$  the net-of-tax rate, optimal earnings are characterised by  $z = g'(z)$ . This is independent of  $\bar{c}$ .

<sup>9</sup>Whilst [Layard \(2006\)](#) focuses the discussion on income comparisons, the framework is static and so income equals consumption. The author recognises that further empirical work is required to differentiate between income and consumption comparisons (see [Layard, 2006](#), p.26, footnote 6).

<sup>10</sup>[Hopkins \(2008\)](#) explores the implications that relative income models generate for inequality studies.

<sup>11</sup>For a study of positive externalities and the excess burden associated with cooperative behaviour see [De Bartolome \(1999\)](#).

and Johansson-Stenman (2010) extend the framework to include within-generation comparisons and upward comparisons (the reference consumption level for all individuals is the average consumption of the high-ability types). The latter case implies higher tax rates on the high ability to reduce the externality they generate.<sup>12</sup> Going beyond atemporal externalities, comparisons with other people’s past consumption also tend to increase marginal tax rates (Aronsson and Johansson-Stenman, 2014).

There is also some work that explores income and commodity taxation when a subset of commodities are subject to relative consumption concerns. Eckerstorfer (2014) analyses a two-ability type model with a nonlinear tax on income and proportional taxes on three commodities, two of which are subject to relative consumption concerns: given the restriction to proportional commodity taxes each tax instrument is required to internalise the externalities associated with relative consumption concerns.

The analysis of relative consumption also relates more generally to that of positional goods. Frank (1985) demonstrates that individuals over-consume positional goods in pursuit of higher rank, but because changes in rank yield no social benefit they are a pure externality. The policy prescription is thus taxation of positional goods vis-à-vis non-positional goods.

Finally, a macroeconomics literature explores tax policy and asset pricing when individual preferences exhibit *Keeping up with the Joneses* behaviour (Ljungqvist and Uhlig, 2000).<sup>13</sup>

## 1.2 Relative Income and Social Welfare

Should relative consumption concerns be considered in social welfare? A number of authors discuss whether it is appropriate to incorporate relative consumption into the social welfare function, particularly if this results in some individuals being made materially worse-off for the subjective well-being of others (Kanbur and Tuomala, 2013; Piketty and Saez, 2012). This is essentially a question about whether government policy objectives should be ‘welfarist’ or ‘paternalistic’. Concerning the latter case, a potential danger with simply dismissing relative consumption concerns as ‘envy’ is that economic policy misses one of the arguments that really matter to people; and is thus not directly aligned with well-being. Furthermore, there are a number of reasons people may care about relative consumption, many of which need not be viewed as ‘negative attitudes’ such as envy (see also Micheletto, 2010). For example, individuals may wish to assess how their lives are going and in doing so naturally use the achievements of others as some form of benchmark. We take the view that, in light of the mounting empirical evidence on well-being and relative income/consumption, analyses which study how policy implications are adjusted in the presence of such concerns are important. The question of whether people

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<sup>12</sup>The tax schedules must satisfy incentive compatibility (self-selection) constraints to avoid high-ability types mimicking low ability types.

<sup>13</sup>See also Wendner (2014, 2015).



should care about relative consumption is of course a different one.

Furthermore, whilst welfarist and paternalistic governments have different objectives - with the latter ‘laundering’ what are seen as undesirable preferences - the optimal tax outcomes may be similar. Indeed, [Aronsson and Johansson-Stenman \(2017\)](#) demonstrate that a paternalistic government’s attempt to correct the individual behavioural ‘failures’ caused by relative consumption concerns generates tax schedules analogous to those arising under a welfarist government that attempts to correct the externalities that arise from consumption comparisons.

The remainder of this paper is structured as follows: Section 2 sets out the general framework and derives our key well-being results; Section 3 derives the optimal income tax expression; Section 4 presents the numerical results; Section 5 provides a discussion of an extension to upwards comparisons; and finally Section 6 concludes the paper. Key proofs and derivations are situated in the Appendix.

## 2 General Framework: Model Setup

Consider an economy with the following properties:

- (i) *Population.* There is a population of individuals of size 1. Individuals differ in their productive ability,  $n$ , as distributed with density function  $f(n)$  satisfying  $f(n) > 0 \forall n > 0$  and  $\int_0^\infty f(n)dn = 1$ . The fraction of individuals at any given productivity is thus given by  $F(n) = \int_0^n f(s)ds$ , where  $F(n) \in [0, 1]$ .
- (ii) *Preferences.* Individuals have homogenous preferences represented by  $u(c, l, \bar{c})$ , where  $c \geq 0$  is consumption;  $l \in [0, 1]$  is the fraction of time spent working<sup>14</sup>; and  $\bar{c}$  is the average consumption of some reference group (to be later specified). We place the following standard assumptions on the utility function:

$$\forall \bar{c} : u_c > 0, u_l < 0, u_{\bar{c}} < 0, u_{cc} < 0, u_{ll} < 0, u_{cc}u_{ll} - u_{cl}^2 > 0. \quad (1)$$

Individual utility is thus increasing in consumption, decreasing in labour, decreasing in the reference consumption level (negative externality); and strictly concave in consumption and labour. In addition, we assume that both consumption and leisure are normal goods:

$$\forall \bar{c} : u_c u_{ll} - u_l u_{cl} > 0, u_l u_{cc} - u_c u_{cl} > 0. \quad (2)$$

- (iii) *The tax-benefit system.* There is a linear income tax system in place under which gross earnings,  $nl$ , are taxed at the rate  $(1 - \alpha)$ , where  $\alpha \in (0, 1)$  is the net-of-tax rate, and

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<sup>14</sup>Such that  $1 - l$  is leisure.



all individuals receive a tax-free basic income,  $\sigma \geq 0$ .<sup>15</sup> Linear income taxation captures the equity-efficiency trade-off of income taxation more tractably than the more general nonlinear taxation, and as such is afforded much analytical attention (Atkinson, 1995; Piketty and Saez, 2012; Viard, 2001). This simplification helps us bring out clearly the effects at work when individual well-being depends on reference consumption, and in turn how these adjust optimal tax considerations.<sup>16</sup>

## 2.1 Individual choices conditional on reference consumption.

Given the budget constraint  $c \leq (n\alpha)l + \sigma$ , individuals choose their labour supply *taking as given* the level of reference consumption,  $\bar{c}$ . The Marshallian labour supply, consumption and resulting indirect utility functions are thus

$$l_M(n\alpha, \sigma, \bar{c}) = \text{Arg max}_{l \in [0,1]} u[(n\alpha)l + \sigma, l, \bar{c}] \quad (3)$$

$$c_M(n\alpha, \sigma, \bar{c}) = n\alpha l_M + \sigma \quad (4)$$

$$v_M(n\alpha, \sigma, \bar{c}) = u(c_M, l_M, \bar{c}) \quad (5)$$

Marshallian labour supply is therefore characterised by

$$n \leq -\frac{u_l[(n\alpha)l_M + \sigma, l_M, \bar{c}]}{u_c[(n\alpha)l_M + \sigma, l_M, \bar{c}]} \cdot \left(\frac{1}{\alpha}\right) \quad ; \quad l_M \geq 0. \quad (6)$$

where the pair of inequalities hold with complementary slackness.

We let  $\underline{n}(\alpha, \sigma, \bar{c}) := -u_l(\sigma, 0, \bar{c})/u_c(\sigma, 0, \bar{c})$  be the reservation productivity satisfying

$$l_M(n\alpha, \sigma, \bar{c}) = 0 \quad \forall n \leq \underline{n} \quad \text{but } l_M > 0 \quad \text{otherwise.} \quad (7)$$

From (6) we can readily distinguish between two cases:

- (i) **Pure Negative Externality.** If  $-u_l/u_c$  is independent of  $\bar{c}$  then so too is  $l_M$ . In this case consumption comparisons are a *pure negative externality*: an increase in  $\bar{c}$  lowers well-being because  $u_{\bar{c}} < 0$  but has no effect on labour supply.
- (ii) **Externality & Keeping up with the Joneses.** Alternatively, if  $-u_l/u_c$  is a function of  $\bar{c}$  then  $l_M$  is a function of  $\bar{c}$ . In line with empirical evidence we assume that  $\partial l_M / \partial \bar{c} > 0$  (which formally requires  $u_c u_{l\bar{c}} - u_l u_{c\bar{c}} > 0$ ). In this case an increase in  $\bar{c}$  lowers well-being

<sup>15</sup>The decision to let  $\alpha$  denote the net-of-tax rate is made for notational convenience throughout the paper.

<sup>16</sup>A number of papers discuss the potential advantages of linear taxation, namely administrative simplicity and work incentives (Paulus and Peichl, 2009; Peichl, 2014). Further, (Mirrlees, 1971, p.208) referred to the optimality of ‘an approximately linear income-tax schedule’.

because  $u_{\bar{c}} < 0$  and induces individuals to increase their labour supply via the *Keeping up with the Jones* (KUJ) effect.

For  $n > \underline{n}(\alpha, \sigma, \bar{c})$  Marshallian labour satisfies

$$\frac{\partial l_M}{\partial(n\alpha)} = [u_c + l_M(u_c u_{cl} - u_l u_{cc})/u_c] (D^{-1}) \quad (8)$$

$$\frac{\partial l_M}{\partial \sigma} = [(u_c u_{cl} - u_l u_{cc})/u_c] (D^{-1}) \leq 0 \quad (9)$$

$$\frac{\partial l_M}{\partial \bar{c}} = [(u_c u_{l\bar{c}} - u_l u_{c\bar{c}})/u_c] (D^{-1}) \geq 0 \quad (10)$$

where  $D = [2u_{cl} - u_{cc}(u_l/u_c) - u_{ll}(u_c/u_l)](u_l/u_c) > 0$  by the concavity (and so quasiconcavity) of utility.

**Remark.** From (8)  $l_M \approx 0$  implies  $\partial l_M / \partial(n\alpha) > 0$ : this will prove useful later.

The Marshallian consumption function,  $c_M(n\alpha, \sigma, \bar{c})$ , has derivatives

$$\frac{\partial c_M}{\partial(n\alpha)} = \frac{\partial l_M}{\partial(n\alpha)} n\alpha + l_M > 0 \quad , \quad \frac{\partial c_M}{\partial \sigma} = \frac{\partial l_M}{\partial \sigma} (n\alpha) + 1 > 0 \quad , \quad \frac{\partial c_M}{\partial \bar{c}} = \frac{\partial l_M}{\partial \bar{c}} (n\alpha) \geq 0 \quad . \quad (11)$$

Finally, the indirect utility function,  $v_M(n\alpha, \sigma, \bar{c})$ , has the following properties:

$$\frac{\partial v_M}{\partial(n\alpha)} = u_c(c_M, l_M, \bar{c}) l_M \geq 0 \quad , \quad \frac{\partial v_M}{\partial \sigma} = u_c(c_M, l_M, \bar{c}) > 0 \quad , \quad \frac{\partial v_M}{\partial \bar{c}} = u_{\bar{c}}(c_M, l_M, \bar{c}) < 0 \quad . \quad (12)$$

Roy's identity thus holds:  $\partial v_M / \partial(n\alpha) = l_M \cdot \partial v_M / \partial \sigma$ .

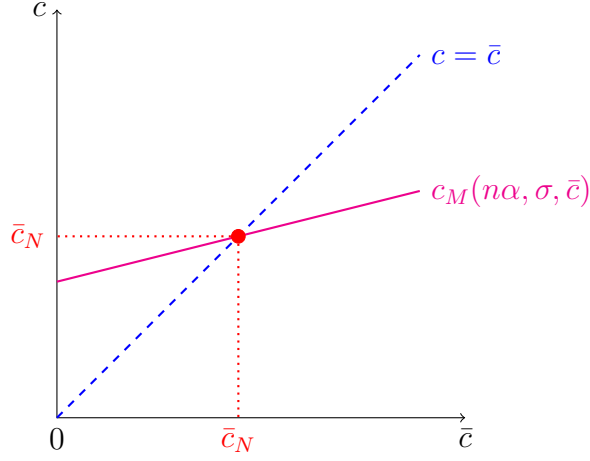
## 2.2 Reference Consumption: Narrow and Broad comparisons.

Individuals may compare their own consumption with those who are similar to them, or more broadly across a wide range of individuals. Empirical evidence typically supports the former (Clark and Senik, 2010; Layard, 2006). For any case the equilibrium levels of reference consumption will be aggregates of individual choices, and so be functions of the tax parameters. We consider two polar cases:

- **Narrow comparison.** In this first case we assume that individuals compare their consumption level with those of the same productivity. Since these individuals have identical preferences, the level of consumption that is commonly chosen in response to a given peer level of consumption will be, in equilibrium, the peer level of consumption. Therefore, for each  $n$ , we can define the *Nash equilibrium* level of consumption,  $\bar{c}_N(n\alpha, \sigma)$ , by the condition that

$$\bar{c}_N(n\alpha, \sigma) \equiv c_M[n\alpha, \sigma, \bar{c}_N(n\alpha, \sigma)] \quad . \quad (13)$$

Figure 1: Equilibrium Reference Consumption (Narrow Case).



**Notes.** This figure graphically illustrates the purpose of assuming  $\partial c_M(n\alpha, \sigma, \bar{c})/\partial \bar{c} < 1$ .

- **Broad comparison.** In this alternative case we assume individuals compare their consumption with the average consumption level across society as a whole. Let  $\bar{c}_B(\alpha, \sigma)$  denote the equilibrium level of average consumption across society: formally this satisfies

$$\bar{c}_B(\alpha, \sigma) \equiv \int_0^\infty c_M[n\alpha, \sigma, \bar{c}_B(\alpha, \sigma)] dF(n) \quad (14)$$

Notice that  $\bar{c}_B$ , being an average, is independent of productivity. This immediately implies that the effect of changes in individual productivity will differ between the Narrow and Broad cases.

**Equilibrium Condition.** We henceforth assume that for any  $(n\alpha, \sigma)$

$$\frac{\partial c_M(n\alpha, \sigma, \bar{c})}{\partial \bar{c}} < 1. \quad (15)$$

This assumption guarantees a unique solution  $\bar{c}_N$  at each productivity level, and is in turn sufficient for a for unique solution  $\bar{c}_B$ . Figure 1 provides graphical intuition for the Narrow (Nash) case.

With this equilibrium condition in place we can immediately establish that the reference consumption levels are multiples greater than 1 of the Marshallian functions:

- For the *Narrow comparison* case

$$\frac{\partial \bar{c}_N}{\partial (n\alpha)} = \frac{\partial c_M}{\partial (n\alpha)} \underbrace{\left(1 - \frac{\partial c_M}{\partial \bar{c}}\right)^{-1}}_{\text{KIJ Multiplier}} > 0, \quad \frac{\partial \bar{c}_N}{\partial \sigma} = \frac{\partial c_M}{\partial \sigma} \underbrace{\left(1 - \frac{\partial c_M}{\partial \bar{c}}\right)^{-1}}_{\text{KIJ Multiplier}}. \quad (16)$$

The effect of a ceteris paribus change in either the gross wage or basic income on Nash consumption is thus an amplification of their effect on Marshallian consumption. Intuitively, an increase in a type  $n$ 's consumption raises peer consumption, in turn causing a type  $n$  to earn more and so forth. Notice that  $(\partial \bar{c}_N / \partial(n\alpha))(\partial \bar{c}_N / \partial \sigma)^{-1} = (\partial c_M / \partial(n\alpha))(\partial c_M / \partial \sigma)^{-1}$ .

- For the *Broad comparison* case

$$\frac{\partial \bar{c}_B}{\partial \alpha} = \frac{\int_0^\infty \frac{\partial c_M}{\partial(n\alpha)} n dF(n)}{\underbrace{\left(1 - \int_0^\infty \frac{\partial c_M}{\partial \bar{c}} dF(n)\right)}_{\text{KUJ Multiplier}}} > 0, \quad \frac{\partial \bar{c}_B}{\partial \sigma} = \frac{\int_0^\infty \frac{\partial c_M}{\partial \sigma} dF(n)}{\underbrace{\left(1 - \int_0^\infty \frac{\partial c_M}{\partial \bar{c}} dF(n)\right)}_{\text{KUJ Multiplier}}} > 0 \quad (17)$$

The effect of the tax parameters on average consumption is thus an amplification of their average effect on Marshallian consumption.

### 2.3 Well-being and the net-of-tax rate.

The indirect utility functions that result under the Narrow and Broad comparator cases are, respectively

$$v_N(n\alpha, \sigma) := v_M[n\alpha, \sigma, \bar{c}_N(n\alpha, \sigma)], \quad (18)$$

$$v_E(n, \alpha, \sigma) := v_M[n\alpha, \sigma, \bar{c}_B(\alpha, \sigma)]. \quad (19)$$

Notice that in the broad case we must distinguish between changes in productivity and changes in the net-of-tax rate, for average consumption across society is independent of the former. The equilibrium levels of labour supply and *individual consumption* that give rise to these indirect utility functions are, respectively

$$l_N(n\alpha, \sigma) := l_M[n\alpha, \sigma, \bar{c}_N(n\alpha, \sigma)] \quad ; \quad c_N(n\alpha, \sigma) := c_M[n\alpha, \sigma, \bar{c}_N(n\alpha, \sigma)] \quad (20)$$

$$l_B(n, \alpha, \sigma) := l_M[n\alpha, \sigma, \bar{c}_B(\alpha, \sigma)] \quad ; \quad c_B(n, \alpha, \sigma) := c_M[n\alpha, \sigma, \bar{c}_B(\alpha, \sigma)] \quad (21)$$

In the Narrow case the definition of individual consumption is trivial because  $c_N = \bar{c}_N$ , but in the Broad case individual consumption differs from the average across society and so the definition of  $c_B$  is important.

From these identities, we can establish the following result.

**Proposition 1.**

(a) *Narrow Case.* Given that

$$\frac{\partial v_N}{\partial(n\alpha)} = u_c(\bar{c}_N, l_N, \bar{c}_N) \left[ l_N - \delta_N \frac{\partial \bar{c}_N}{\partial(n\alpha)} \right] ; \delta_N := -\frac{u_{\bar{c}}(\bar{c}_N, l_N, \bar{c}_N)}{u_c(\bar{c}_N, l_N, \bar{c}_N)} \geq 0 \quad (22)$$

it immediately follows that for Nash reservation productivity  $\underline{n}_N := \underline{n}(\alpha, \sigma, \sigma)$  we have

$$\frac{\partial v_N}{\partial(n\alpha)} \begin{cases} = 0 & : n \leq \underline{n}_N \\ < 0 & : n > \underline{n}_N \text{ but } n \approx \underline{n}_N \\ > 0 & : \text{otherwise} \end{cases} \quad (23)$$

(b) *Broad Case.* Given that  $\partial v_B / \partial n = u_c(c_B, l_B, \bar{c}_B) \alpha l_B$  but

$$\frac{\partial v_B}{\partial \alpha} = u_c(c_B, l_B, \bar{c}_B) \cdot \left[ n l_B - \delta_B \frac{\partial \bar{c}_B}{\partial \alpha} \right] ; \delta_B := \frac{u_{\bar{c}}(c_B, l_B, \bar{c}_B)}{u_c(c_B, l_B, \bar{c}_B)} \geq 0 \quad (24)$$

it immediately follows that for reservation productivity  $\underline{n}_B := \underline{n}(\alpha, \sigma, \bar{c}_B)$

$$\frac{\partial v_B}{\partial n} \begin{cases} = 0 & : n \leq \underline{n}_B \\ > 0 & : n > \underline{n}_B \end{cases} ; \frac{\partial v_B}{\partial \alpha} \begin{cases} < 0 & : n \leq \underline{n}_B \\ < 0 & : n > \underline{n}_B \text{ but } n \approx \underline{n}_B \\ > 0 & : \text{otherwise} \end{cases} \quad (25)$$

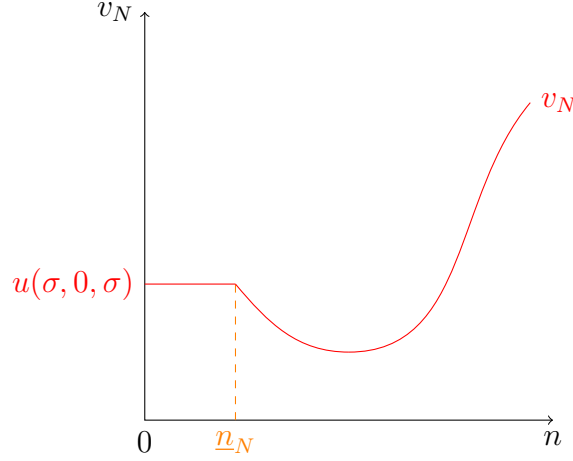
*Proof: See Appendix.*

A number of important messages are contained within Proposition 1. First, in the Narrow case we can capture the effect of changes in individual productivity and the net-of-tax rate via the net wage,  $n\alpha$ ; whereas in the broad case we need to distinguish between changes in individual productivity,  $n$ , and changes in the net-of-tax rate,  $\alpha$ . Proposition 1(a) states that in the Narrow case well-being is decreasing in the net wage rate for workers with productivity close to the Nash reservation productivity. The intuition for this follows from the two opposing effects in expression (22): an increase in the net wage rate (i) increases own-income and so acts to raise well-being at a rate proportional to labour supply (the standard effect by Roy's identity); but also (ii) increases the Nash consumption level, which acts to lower well-being. For workers with productivity close to the reservation productivity the first effect is approximately zero, and so the second negative effect dominates. The immediate implication of Proposition 1(a) is that, in contrast to the conventional theory, the worst-off in society are no longer the voluntarily unemployed or lowest productivity, but instead those of low productivity who work.<sup>17</sup> This is

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<sup>17</sup>Proposition 1(a) represents a *substantial* generalisation to the key result in [Ulph \(2014\)](#): the latter uses a specific functional form to demonstrate that under Narrow comparisons well-being falls with the net wage for workers with wages close to the reservation wage.

Figure 2: Nash Indirect Well-being over the net wage continuum.



**Notes.** This figure graphically illustrates Result 1(a). Nash Indirect well-being is falling in the net - and so gross - wage for workers with  $n \approx \underline{n}_N$ .

illustrated graphically in Figure 2.

Turning to the Broad case, Proposition 1(b) states that whilst well-being is non-decreasing in individual productivity, it is decreasing in the net-of-tax rate for individuals of low productivity (notice that we do not specify *workers* of low productivity). The intuition follows from the two opposing effects in (24). For the voluntarily unemployed an increase in the net-of-tax rate has no effect on own-consumption but does act to raise average consumption, thus lowering well-being. For workers, an increase in the net-of-tax rate (i) increases own-income and so acts to raise well-being at a rate proportional to gross earnings; but also (ii) raises average income across society. For workers with productivity close to the reservation productivity, gross earnings are approximately zero and so the latter effect dominates.<sup>18</sup>

## 2.4 Well-being and the basic income: social marginal welfare weights

In modern tax analysis the social marginal welfare weight (smww) captures the value, in terms of public funds, that society places on a unit increase in a given individual's unearned income (Piketty and Saez, 2012). In the conventional framework the marginal indirect utility of unearned

<sup>18</sup>Part of Proposition 1 can be generalised to a Piketty and Saez (2012) 'style' framework in which individuals differ in their preferences and earnings ability, with heterogeneity captured by the index  $i$ . A type  $i$  individual has preferences  $u^i(c, z, \bar{c})$ , where  $z \geq 0$  is gross earnings. Marshallian gross earnings,  $z_M^i$ , satisfy  $\alpha = -u_z^i/u_c^i$ , and the resulting indirect utility is  $v_M^i$ . Letting  $z_x^i$  and  $v_x^i$ ,  $x = N, B$ ; be the equilibrium outcomes, one can readily demonstrate that

$$z_x^i \approx 0 \Rightarrow \partial v_x^i / \partial \alpha < 0.$$

The strength of this result is that the sign of the inequality has changed relative to the standard theory. However, because we allow heterogeneity in preferences we cannot make interpersonal comparisons and so cannot make all the claims we do in Proposition 1.

income is simply the marginal utility of consumption; and so the smww is proportional to the marginal utility of consumption (and distributional weights as captured by concave transformations of utility). One can readily establish from (18) and (19) that the marginal indirect utility unearned income in our framework is

$$\frac{\partial v_x}{\partial \sigma} = u_c(c_x, l_x, \bar{c}_x) \cdot \left[ 1 - \delta_x \frac{\partial \bar{c}_x}{\partial \sigma} \right] ; x = N, B . \quad (26)$$

Paralleling somewhat our discussion around Proposition 1, (26) illustrates that a ceteris paribus increase in the basic income generates two opposing effects: it (i) increases own-income which acts to raise well-being at a rate proportional to the marginal utility of consumption (the standard Roy's identity effect); but also (ii) raises the equilibrium level of reference consumption. This discussion illustrates how the effect of an increase in the basic income on well-being differs from the standard framework. However, unlike the effect of a change in the net wage/net-of-tax rate - which for some individuals will clearly be well-being reducing by (22) and (24) - there is no such clear case from (26). We henceforth making the following assumption:

**Assumption.**  $\partial v_x / \partial \sigma \geq 0 \forall n$ .<sup>19</sup>

In the optimal tax analysis that follows in Section 3, we will consider a generalised utilitarian social welfare function of the form  $SWF = \int_0^\infty G(v_x) dF(n)$ , where  $G(\cdot)$  is a concave transformation capturing societal concern for inequality in utility/well-being levels.<sup>20</sup> Under such an objective criterion, we can define the smww of a productivity  $n$  individual by

$$g_x(n; \alpha, \sigma) := \frac{G'(v_x) \frac{\partial v_x}{\partial \sigma}}{\lambda_x} ; x = N, B \quad (27)$$

where  $\lambda_x$  is the shadow price of public expenditure (which in the optimal tax analysis will satisfy  $\lambda_x = \int_0^\infty G' \partial v_x / \partial \sigma dF(n)$  and so be independent of  $n$ .)

An important consideration for optimal tax analysis is how the smww changes across individuals, and thus across productivity levels. In the standard framework the smww falls unambiguously with productivity (whilst earnings rise), thus prompting a desire to redistribute from those of high earnings (low smww) to those of low earnings (high smww). Partially differentiating (27) w.r.t.  $n$  and drawing on Proposition 1 allows us to state the following:

**Proposition 2.** *When society cares about inequality in well-being levels (as captured by  $G'' < 0$ ), the smww may fall less rapidly with productivity under Narrow comparisons than under*

<sup>19</sup>While there may be some interesting policy implications that arise through pursuing further the possibility of a non-positive marginal utility of income, we feel that (i) this case is unlikely; and (ii) considering it will reduce the focus of the analysis.

<sup>20</sup>The generalised utilitarian social welfare function is the dominant approach in normative public finance (see Atkinson, 1995; Piketty and Saez, 2012).



Broad comparisons (and relative to the standard theory) because

$$\frac{\partial g_x}{\partial n} = \frac{1}{\lambda_x} \left\{ \left[ G''(v_x) \frac{\partial v_x}{\partial n} \right] \frac{\partial v_x}{\partial \sigma} + G'(v_x) \frac{\partial^2 v_x}{\partial \alpha \partial \sigma} \right\} ; x = N, B \quad (28)$$

From Proposition 1 we know that in the Narrow comparison case  $\partial v_N / \partial(n\alpha) < 0$  for workers with productivities close to the reservation wage, such that the term in square braces in (28) is overall positive. Contrastingly, in the Broad comparison case we have  $\partial v_B / \partial n > 0$  unambiguously such that the term in square braces is unambiguously negative, in line with the conventional theory. The sign of the second term in (28) is ambiguous.<sup>21</sup>

In conventional optimal tax analysis, one employs Roy's identity - the relationship between the marginal indirect utility of the net wage and the marginal indirect utility of unearned income - to write the optimal tax expression in terms of the smww. The discussion around Propositions 1 and 2 has made clear that the conventional Roy's identity does not apply here. To facilitate the analysis in Section 3, we thus derive what we term the 'distortion-adjusted' Roy's identity.

**Lemma 1. (Distortion-Adjusted Roy's identity).**

- (a) *Narrow Case*

$$\frac{\partial v_N(n\alpha, \sigma)}{\partial(n\alpha)} \cdot n = \frac{\partial v_N(n\alpha, \sigma)}{\partial \sigma} \cdot (nl_N - \Delta_N) ; \quad \Delta_N := \frac{n\delta_N \frac{\partial \bar{c}_{HN}}{\partial(n\alpha)}}{1 - \delta_N \frac{\partial \bar{c}_N}{\partial \sigma}} \quad (29)$$

where  $\partial \bar{c}_{HN} / \partial(n\alpha)$  is the compensated effect of an increase in the net wage on Nash consumption.

- (b) *Broad Case*

$$\frac{\partial v_B(n, \alpha, \sigma)}{\partial \alpha} = \frac{\partial v_B(n, \alpha, \sigma)}{\partial \sigma} \cdot (nl_B - \Delta_B) ; \quad \Delta_B := \frac{\delta_B \left[ \frac{\partial \bar{c}_B}{\partial \alpha} - nl_B \frac{\partial \bar{c}_B}{\partial \sigma} \right]}{1 - \delta_B \frac{\partial \bar{c}_B}{\partial \sigma}} \quad (30)$$

*Proof: See Appendix.*

We refer to the terms  $\Delta_x$  as the 'distortion-adjustments' that are applied to the conventional Roy's identity.

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<sup>21</sup>In the strict utilitarian case  $G'' = 0$  and so the first term in (28) will be zero because there is no concern for inequality in well-being levels.

## 2.5 Inefficiency of Narrow and Broad Equilibria.

To provide the intuition for these distortion-adjustment terms, it is insightful to consider what the socially optimum labour supply function is for both the Narrow and Broad comparison cases. In the Narrow (Nash) case, things are certainly clear: were individuals to recognise the interplay between their own choices and that of their peers, they would behave differently. Yet, in the Broad case individual choices have effectively no impact on the average consumption level across society, and so we must appeal to a different, Kantian, criterion for a social optimum. These arguments are made clear in the following Result.

**Proposition 3. (Inefficiency of Narrow and Broad Equilibria.)**

- (a) **Narrow (Nash) Case.** *Taking the tax system as given, a social planner would recognise that in the Nash Equilibrium  $c = \bar{c}$  and so choose  $l_{SN}(n\alpha, \sigma) := \text{Arg max } u(n\alpha l + \sigma, l, n\alpha l + \sigma)$ , where the social optimum labour supply function,  $l_{SN}$ , is characterised by*

$$n\alpha \left( 1 + \frac{u_{\bar{c}}}{u_c} \right) \leq -\frac{u_l}{u_c} ; l_{SN} \geq 0 \quad (31)$$

*Comparing this with the Nash equilibrium outcome of  $n\alpha \leq u_l/u_c$  ,  $l_N \geq 0$ ; we can see that  $l_N \neq l_S$  and so Nash labour supply is inefficient. This ‘distortion’ can in principle be corrected by a (productivity-specific) Pigovian tax on earnings set at  $-u_{\bar{c}}/u_c$  (evaluated at  $l_{SN}$ ).*

- (b) **Broad Case.** *Letting  $c_n(l) = n\alpha l + \sigma$ , one can define the Broad social optimum labour supply function via the Kantian conjectural variation that a change in own-consumption generates a corresponding change in average consumption. Formally*

$$\begin{aligned} l_{SB}(n, \alpha, \sigma) &:= \text{Arg max}_{l \in (0,1)} u [c_n(l), l, \bar{c}] \\ \text{s.t. } dc &= d\bar{c} \quad (\text{Kantian Conjectural Variation}) ; \\ \bar{c}_{SB} &= \int_0^\infty c_n(l_{SB}) dF(n) . \end{aligned} \quad (32)$$

*In this case  $l_{SB}$  is characterised by an expression analogous to (31).*

*Proof: See Appendix.*

Proposition 3 can be interpreted as follows. In the Narrow (Nash) case, were individuals to recognise that in equilibrium their consumption will coincide with that of their peers, they would make different choices relative to those made under the Nash conjectural variation. Indeed, the second term on the left side of (31) makes clear that individuals internalise the externality of consumption comparisons when making their choices. Notice that an alternative interpretation of the Narrow social optimum is that it is the result of *Kantian* decision making: a productivity

$n$  individual chooses the earnings level that they would also advocate for all other productivity  $n$  individuals, thus maximising their own well-being and everybody else like them! A key feature of this interpretation is that we move from a Nash conjectural variation (self-interested calculus), to what we might think of as a *Kantian conjectural variation*: if I change my own consumption by 1 unit so will everybody else. This thought process allows us to define a social optimum in the broad case.

Let the indirect utility functions under the Narrow and Broad *Socially* optimal choices be

$$v_{SN}(n\alpha, \sigma) := (n\alpha l_{SN} + \sigma, l_{SN}, n\alpha l_{SN} + \sigma) \quad (33)$$

$$v_{SB}(n, \alpha, \sigma) = (n\alpha l_{SB} + \sigma, l_{SB}, \bar{c}_{SB}) \quad (34)$$

From (33) and (34) one can readily demonstrate that

$$\frac{\partial v_{SN}}{\partial(n\alpha)} = \frac{\partial v_{SN}}{\partial\sigma} l_{SN} \ ; \ \frac{\partial v_{SB}}{\partial\alpha} = \frac{\partial v_{SB}}{\partial\sigma} n l_{SB} \quad (35)$$

A comparison between (35) and the distortion-adjusted Roy's identity in (29) and (30) makes clear why we refer to the terms  $\Delta_x$  as the distortion-adjustments.

We are now in a position to derive the optimal income tax expressions for both the Narrow and Broad cases.

### 3 Optimal Income Taxation

In this section we derive a general expression that characterises the optimal linear income tax under both Narrow and Broad comparator cases. In line with modern tax analysis (see [Piketty and Saez, 2012](#)), our approach will use *weighted* smwvs and elasticities to characterise the optimal tax rate.

Let  $z_x := n l_x$ ,  $x = N, B$ , be the gross earnings of a productivity  $n$  individual in either the Narrow or Broad comparison cases. Aggregate gross earnings are thus denoted by

$$Z_x(\alpha, \sigma) := \int_0^\infty z_x dF(n) \ ; \ x = N, B \quad (36)$$

The (purely redistributive) government budget constraint is therefore:<sup>22</sup>

$$\sigma \leq (1 - \alpha) Z_x(\alpha, \sigma)$$

The basic income level that balances the government budget (for any tax rate) is

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<sup>22</sup>We pursue the purely redistributive case solely for analytical simplicity and ease of notation: no great additional insight is attained through imposing an exogenous revenue requirement for spending outside of welfare (the lump-sum benefit).

$$\sigma_x(1 - \alpha) \equiv (1 - \alpha) \cdot \tilde{Z}_x(\alpha) \quad ; \quad \tilde{Z}_x := Z_x[\alpha, \sigma_x(1 - \alpha)] \quad (37)$$

The conventional results apply:  $d\sigma_x/d(1-\alpha) = \tilde{Z}_x - (1-\alpha)d\tilde{Z}_x/d\alpha$  and so the revenue maximising net-of-tax rate,  $\alpha_L$ , is characterised by

$$\frac{1 - \alpha_L}{\alpha_L} = \frac{1}{e_x} \quad \text{where} \quad e_x := \frac{\alpha}{\tilde{Z}_x} \frac{d\tilde{Z}_x}{d\alpha} \quad (38)$$

is the elasticity of equilibrium aggregate earnings w.r.t. the net-of-tax rate.

**Optimal income tax problem.** The optimal linear income tax problem is thus

$$\hat{\alpha}_x \equiv \arg \max_{\alpha \in (0,1)} \int_0^\infty G(v_x) dF(n) \quad \text{s.t.} \quad \sigma = \sigma_x(1 - \alpha) \quad ; \quad x = N, B \quad (39)$$

As discussed in Section 2.4, the function  $G$  is a concave transformation of individual well-being that captures societal concern for inequality in well-being levels.

To proceed, we define the following two terms:

$$\bar{g}_x^z := \int_0^\infty g_x \left( \frac{z_x}{\tilde{Z}_x} \right) dF(n) \quad (40)$$

$$\bar{g}_x^\Delta := \int_0^\infty g_x \left( \frac{\Delta_x}{\tilde{Z}_x} \right) dF(n) \quad (41)$$

where the definition of  $g_x$  follows from (27); with  $\lambda_x = \int_0^\infty G'(v_x) \partial(v_x/\partial\sigma) dF(n)$ . As found in some form in standard tax analysis,  $\bar{g}_x^z$  is the average of the individual smww weighted by relative gross earnings (Piketty and Saez, 2012). Meanwhile,  $\bar{g}_x^\Delta$  is the average of the individual smww weighted by the distortion-adjustment relative to gross earnings.

**Proposition 4. (Optimal Linear Income Tax).** *The implicit expression characterising the optimal income tax for both Narrow and Broad comparison cases is*

$$\frac{1 - \hat{\alpha}_x}{\hat{\alpha}_x} = \frac{\bar{g}_x^\Delta + (1 - \bar{g}_x^z)}{e_x} \quad ; \quad x = N, B \quad (42)$$

where  $1 - \bar{g}_x^z = -\text{Cov}(g_x, z_x/\tilde{Z}_x)$ .

*Proof:* See Appendix.

We can see from (42) that, in contrast to the standard optimal linear income tax expression, there are now *three* considerations in setting the optimal tax rate: (i) distortion/externality correction; (ii) the negative covariance between the smww and relative gross earnings (standard equity considerations); and (iii) the traditional aggregate earnings elasticity (efficiency considerations). We discuss each in turn.

- (i) *Distortion/Externality correction.* The first term in the numerator of (42),  $\bar{g}_x^\Delta$ , captures the fact that reference consumption is a negative externality that individuals (i) do not account for in the Narrow case due to Nash behaviour; or (ii) do not influence in the Broad case. We have demonstrated that in both comparator cases there are social optima in which individual's internalise this externality in some form, thus resulting in a 'distortion' between the equilibrium choices and the socially optimum choices. Ceteris paribus, the optimal tax rate rises to partially correct these distortions on average. This is analogous to the argument for higher tax rates proposed in the existing literature ([Boskin and Sheshinski, 1978](#); [Layard, 2006](#); [Ljungqvist and Uhlig, 2000](#)).
- (ii) *Covariance between the smww and relative gross earnings.* The second term in the numerator of (42),  $1 - \bar{g}_x^z$ , is the negative of the covariance between the smww and relative gross earnings. In the standard theory this covariance is unambiguously negative, thus capturing a desire to redistribute from those of high gross earnings (low smww) towards those of low gross earnings (high smww). In our framework, Results 1 and 2 have demonstrated that (a) the smww differs from the standard theory; and importantly (b) the smww may fall less rapidly with productivity under Narrow comparisons than under Broad comparisons, and indeed relative to the standard theory. To the extent that this holds, the redistributive motives embodied in this covariance may be somewhat lessened in the Narrow case.
- (iii) *Aggregate Elasticity.* At this level of generality, there is not too much we can say about the aggregate elasticity,  $e_x$ , in the denominator of (42). In the context of our framework, the presence of *KUJ* effects renders equilibrium labour supply more responsive to the tax parameters than predicted by the conventional framework, and this will therefore affect the aggregate elasticity. In reality of course, we can estimate aggregate elasticities and so from a practical perspective it is unclear how much the theory adds in this regard.

## 4 Numerical Analysis

To gain more understanding into how the different considerations in (42) affect optimal tax rates, we turn to numerical methods. We view a desirable functional form for the numerical analysis as one which can capture

- (i)  $\bar{c}$  as a pure negative externality; or
- (ii)  $\bar{c}$  as both a negative externality and an argument which increases labour supply via *KUJ* effects; and
- (iii) the traditional model as a special case in which individuals place no weight on income comparisons.

In line with modern tax analysis, we consider an augmentation of the frequently employed isoelastic utility function (see [Atkinson, 1990](#); [Saez, 2001](#)):

$$u = \log \left[ \underbrace{\left( c - \frac{l^{1+k}}{1+k} \right)}_{\text{Isoelastic utility}} - \underbrace{\theta \cdot \left( \frac{\bar{c}}{1 + \chi \cdot c} \right)}_{\text{'Relative' consumption}} \right] ; \quad \theta \geq 0, \chi \geq 0 \quad (43)$$

The parameter  $\theta \geq 0$  captures the weight that individuals place on consumption comparisons. One can show that  $u_{c\theta} \geq 0$ , such that the marginal utility of consumption is increasing in the weight individuals place on consumption comparisons. If  $\theta = 0$  the preferences reduce to the standard isoelastic utility function, where  $1/k$  is the labour elasticity. However, if  $\theta > 0$  we have  $u_{\bar{c}} < 0$  and the parameter  $\chi \geq 0$  determines whether or not  $\bar{c}$  is an argument of labour supply:

- $\chi = 0$ : In this case  $\partial l_M / \partial \bar{c} = 0$  and so  $\bar{c}$  is a *pure negative externality*.
- $\chi > 0$ : In this case  $\partial l_M / \partial \bar{c} > 0$  and so  $\bar{c}$  is both a negative externality and an argument of labour supply. Indeed, one can readily demonstrate that  $u_c u_{l\bar{c}} - u_l u_{c\bar{c}} > 0$ , thus satisfying the KUJ requirement in (10).

To avoid negative utility in the computations we can always add a positive constant.

The parameter  $\theta$  can be related to our theoretical analysis as follows: in the context of this functional form our theoretical analysis is essentially comparing the case with  $\theta > 0$  ( $u_{\bar{c}} < 0$ ) with  $\theta = 0$  ( $u_{\bar{c}} = 0$  - the standard model). Our theoretical discussion does not, however, say much about the size of relative consumption concerns, and so the size of  $u_{\bar{c}}$ . A key purpose of the numerical analysis is therefore to explore how optimal tax rates change when the weight that individuals place on relative consumption changes (note that  $u_{\bar{c}\theta} < 0$ ).

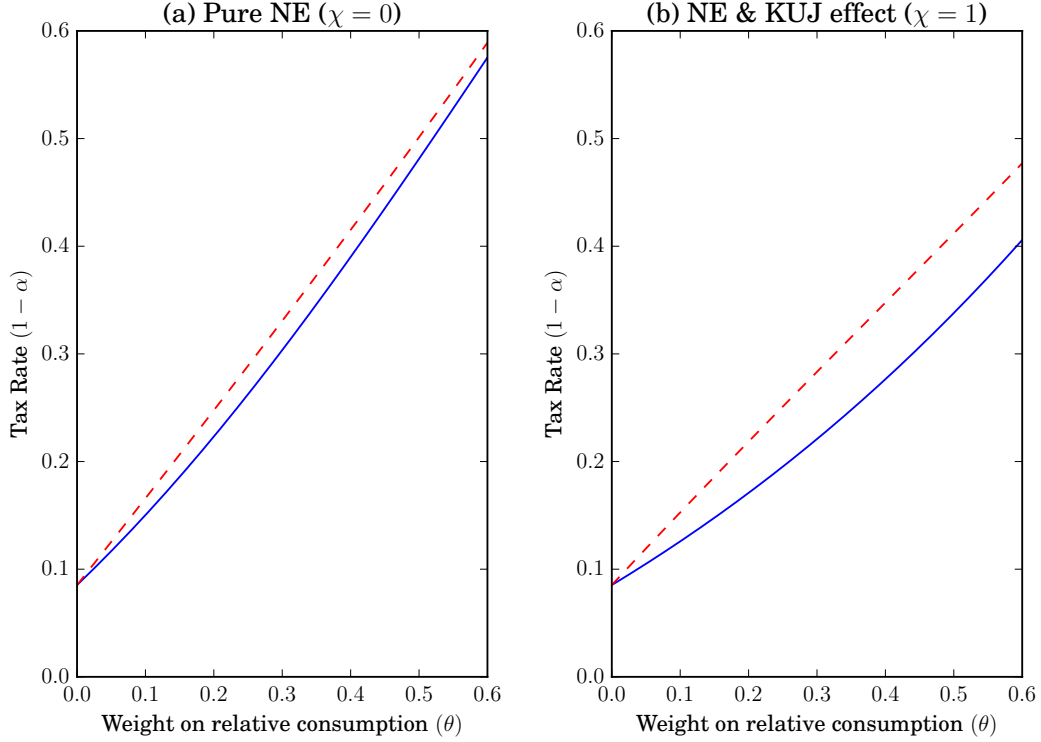
Whilst one could choose a number of different functional forms for the analysis, this particular form has the virtue that it (i) embeds the isoelastic utility function; and - in line with our theoretical analysis - (ii) can unambiguously generate KUJ effects on labour supply.<sup>23</sup>

**Parameter choices and numerical procedure.** For simplicity, we assume that  $n$  is lognormally distributed where  $\log(n)$  has mean -1 and standard deviation 0.39. We set  $k = 2$ , and consider  $\chi \in \{0, 1\}$  and  $\theta \in [0, 0.6]$ . We vary  $\theta$  in increments of 0.025, at each stage simulating the optimal tax rate that maximises the social welfare function  $\int_0^\infty u \, dF(n)$  subject to the government budget constraint (see Appendix for the numerical code).<sup>24</sup>

<sup>23</sup>[Kanbur and Tuomala \(2013, p.1208\)](#) use the functional form  $u(c, l, \bar{c}) = \log c + \theta \log(c/\bar{c}) + \log(1 - l)$ . Under these preferences labour supply is characterised by the first-order condition  $(n\alpha) \leq (n\alpha l_M + \sigma)(1 + \theta)/(1 - l_M)$ . So whilst labour supply is a function of the *weight* individuals place on relativity concerns, it is *independent* of the peer consumption level. There are thus no *KUJ* effects.

<sup>24</sup>Note that we could have specified preferences without the *log* transformation and instead included a  $G = \log(u)$  in the social welfare function.

Figure 3: Simulated Optimal Tax Rates: Narrow and Broad comparator cases.



**Numerical results.** The numerical results are presented in Figure 3: panel (a) presents results for the case where reference consumption is a pure negative externality ( $\chi = 0$ ), whilst panel (b) presents the case where reference consumption is both a negative externality and an argument of labour supply ( $\chi = 1$ ). The two panels have the same structure: on the horizontal axis  $\theta$  is varied from 0 to 0.6, whilst the vertical axis displays the tax rate,  $1 - \alpha$ . The unbroken line in both panels is the optimal tax rate under Narrow comparisons, whilst the dashed line is the optimal tax rate under Broad comparisons.

We can make a number of key observations from Figure 3. First, in each case the optimal tax rate rises with  $\theta$ , and thus with the weight that individuals place on consumption comparisons. The intuition would seem to follow from the  $\bar{g}_x^\Delta$  term in (42). Ceteris paribus, the fact that  $\partial^2 u / \partial \theta \partial \bar{c} < 0$  in (43) suggests that the size of the externality/distortion increases with  $\theta$ , thus prompting higher tax rates on distortion-correction grounds. Second, a comparison between panels (a) and (b) illustrates that optimal tax rates are lower in the presence of *KUJ* effects than otherwise. The intuition here may be two-fold: to the extent that *KUJ* effects render earnings more responsive to the tax rate (i) conventional tax theory would suggest that redistributive aims are more constrained, thus acting to lower tax rates; and further (ii) distortion-correction may be achieved through smaller changes in the tax rate. The latter argument illustrates that



the interaction between distortion-correction and the aggregate elasticity should not be seen as a trade-off. Third, in both panels the optimal tax rate under Broad comparisons everywhere exceeds that under Narrow comparisons. The intuition here may follow in part from Results 1 and 2, and thus the  $1 - \bar{g}_x^z$  term in the numerator of (42). To the extent that the smww falls less rapidly with productivity under Narrow comparisons than under Broad comparisons, this may act to lower the optimal tax rate in the Narrow case relative to the Broad case.<sup>25</sup>

## 5 Upwards comparisons

In the main analysis we have assumed that individuals either compare their consumption with those of the same productivity, or against the average consumption in society. An alternative reference case is that of upwards comparisons, whereby individuals only compare their consumption with those whose productivity exceeds their own. The purpose of this section is to briefly discuss how our key well-being results are likely to persist in this alternative case.<sup>26</sup>

In the case of upwards comparisons let  $\bar{c}_U(n, \alpha, \sigma)$  be the reference consumption of a productivity  $n$  individual: this is the average consumption of all individuals with productivity  $s > n$ , as implicitly defined by the system of equations

$$\bar{c}_U(n, \alpha, \sigma) = \frac{\int_n^\infty c_M[s\alpha, \sigma, \bar{c}_U(s, \alpha, \sigma)] dF(s)}{1 - F(n)} \quad (44)$$

We assume that (i) there is a unique solution to this system of equations; and (ii) there is still a unique reservation productivity.

An inspection of Proposition 1 suggests that our well-being result will have a natural analogue in the upwards comparison case if  $\partial \bar{c}_U / \partial n > 0$ , for the increase in reference consumption resulting from an increase in the wage will offset the conventional effect for low productivity individuals.<sup>27</sup> Differentiating (44) w.r.t.  $n$  yields

$$\frac{\partial \bar{c}_U}{\partial n} = \frac{f(n)}{[1 - F(n)]^2} \int_n^\infty \{c_M[s\alpha, \sigma, \bar{c}_U(s, \alpha, \sigma)] - c_M[n\alpha, \sigma, \bar{c}_U(n, \alpha, \sigma)]\} dF(s) \quad (45)$$

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<sup>25</sup>The nature of these discussions is naturally speculative, resulting from the fact optimal tax rates are *implicitly* characterised: terms on the right side of (42) are themselves functions of the tax rate.

<sup>26</sup>This section is not intended to provide any formal proofs, but rather a first inspection of upwards comparisons.

<sup>27</sup>Formerly, if we let  $v_U(n, \alpha, \sigma) := v_M[n\alpha, \sigma, \bar{c}_U(n, \alpha, \sigma)]$  and  $l_U(n, \alpha, \sigma) := l_M[n\alpha, \sigma, \bar{c}_U(n, \alpha, \sigma)]$  then

$$\frac{\partial v_U}{\partial n} = u_c \left\{ \alpha l_U + \frac{u_{\bar{c}}}{u_c} \frac{\partial \bar{c}_U}{\partial n} \right\}.$$

So long as there a unique reservation productivity such that labour supply is either zero or very low for low productivity individuals, well-being will decrease with productivity for those of sufficiently low productivity provided  $\partial \bar{c}_U / \partial n > 0$ .

The terms within curly braces in (45) can be written more insightfully as

$$\begin{aligned}
& c_M[s\alpha, \sigma, \bar{c}_U(s, \alpha, \sigma)] - c_M[n\alpha, \sigma, \bar{c}_U(n, \alpha, \sigma)] \\
&= \underbrace{\{c_M[s\alpha, \sigma, \bar{c}_U(s, \alpha, \sigma)] - c_M[n\alpha, \sigma, \bar{c}_U(s, \alpha, \sigma)]\}}_{>0} \\
&+ \{c_M[n\alpha, \sigma, \bar{c}_U(s, \alpha, \sigma)] - c_M[n\alpha, \sigma, \bar{c}_U(n, \alpha, \sigma)]\}
\end{aligned} \tag{46}$$

The first term on the right side of the equality in (46) compares the consumption of a type  $s$  individual with a type  $n$  individual when both have the same reference group: this is unambiguously positive because we know from (11) that  $\partial c_M / \partial \bar{c} > 0$ . The second term compares how a type  $n$  individual's consumption evaluated at the reference consumption of a type  $s$  individual differs from that evaluated at their own reference consumption: if  $\bar{c}_U(s, \alpha, \sigma) > \bar{c}_U(n, \alpha, \sigma)$  this term is positive

If we assert that  $\partial \bar{c}_U / \partial n > 0$  then the right side of (3) - and so (2) - is unambiguously positive, thus consistent with the assertion. However, if we alternatively assert that  $\partial \bar{c}_U / \partial n < 0$  then the sign of (3) is ambiguous: it is only negative if the second term offsets the first, which seems a stringent condition and requires cases where  $\partial c_M / \partial \bar{c} > \partial c_M / \partial n \alpha$ .

## 6 Concluding Remarks

This paper has sought to precisely draw out the behavioural, well-being and in turn optimal tax implications that arise when individual well-being is decreasing in the consumption of some reference group. We considered two polar cases: individuals may compare their consumption with those of the same productivity (the Narrow case) or the average consumption across society (the Broad case).

To the best of our knowledge, this paper is the first to demonstrate - under highly general preferences - that well-being decreases with the net wage (net-of-tax) rate under Narrow (Broad) comparisons for workers of low productivity. This arises because the conventional well-being effect (via Roy's identity) is offset by the externality effect of increasing reference consumption for these individuals. This has implications for how social marginal welfare weights change with productivity.

The externality of reference consumption distorts individual choices away from the social optimum. Whilst the social optimum is well-documented for cases equivalent to the Narrow case - where a planner recognises that in equilibrium own consumption equals peer consumption - the Broad case has received little attention.<sup>28</sup> We characterise the Broad social optimum using Kantian calculus: individuals choose labour supply under the Kantian conjectural variation that

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<sup>28</sup>Layard (2006) derives the socially optimum labour supply for a population of *identical* individuals.

a change in own-consumption generates the same change in reference consumption. The Kantian equilibrium concept is increasingly employed in the study of externalities and generates a social optimum expression analogous to the Narrow case.

To tractably pin-down these well-being and behavioural implications, we have restricted the analysis to a linear income tax. This serves to illustrate well how conventional tax considerations are adjusted in the presence of relative consumption concerns. An important future contribution will be to undertake the analysis in a fully nonlinear context.

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# Appendices

## A Derivations and Proofs

### Derivation of Proposition 1

• *Proposition 1a*

$$\begin{aligned}
\frac{\partial v_N(n\alpha, \sigma)}{\partial(n\alpha)} &= \frac{\partial v_M(n\alpha, \sigma, \bar{c}_N)}{\partial(n\alpha)} + \frac{\partial v_M(n\alpha, \sigma, \bar{c}_N)}{\partial \bar{c}} \frac{\partial \bar{c}_N}{\partial(n\alpha)} \\
&= u_c(\bar{c}_N, l_N, \bar{c}_N) \cdot \left[ l_N + \frac{u_{\bar{c}}(\bar{c}_N, l_N, \bar{c}_N)}{u_c(\bar{c}_N, l_N, \bar{c}_N)} \frac{\partial \bar{c}_N}{\partial(n\alpha)} \right] \\
&= u_c(\bar{c}_N, l_N, \bar{c}_N) \cdot \left[ l_N - \delta_N \frac{\partial \bar{c}_N}{\partial(n\alpha)} \right]
\end{aligned} \tag{A.1}$$

where  $\delta_N = -u_{\bar{c}}(\bar{c}_N, l_N, \bar{c}_N)/u_c(\bar{c}_N, l_N, \bar{c}_N)$ .

For  $n \leq \underline{n}$  we have both  $l_N = 0$  and  $\partial \bar{c}_N / \partial(n\alpha) = 0$  and so from (A.1)  $\partial v_N / \partial(n\alpha) = 0$ . Contrastingly, for  $n > \underline{n}_N$  but  $n \approx \underline{n}_N$  we have (a)  $l_N \approx 0$  by the continuity of the labour supply function; and (b)  $\partial \bar{c}_N / \partial(n\alpha) > 0$  by (8), (11) and (16). In this case it follows from (A.1) that  $\partial v_N / \partial(n\alpha) \approx u_c \delta_N \partial \bar{c}_N / \partial(n\alpha) < 0$ . ■

• *Proposition 1b*

$$\begin{aligned}
\frac{\partial v_B(n, \alpha, \sigma)}{\partial n} &= \frac{\partial v_M[n\alpha, \sigma, \bar{c}_B(\alpha, \sigma)]}{\partial(n\alpha)} \alpha \\
&= u_c(c_B, l_B, \bar{c}_B) \alpha l_B \geq 0
\end{aligned} \tag{A.2}$$

$$\begin{aligned}
\frac{\partial v_B(n, \alpha, \sigma)}{\partial \alpha} &= \frac{\partial v_M[n\alpha, \sigma, \bar{c}_B(\alpha, \sigma)]}{\partial(n\alpha)} n + \frac{\partial v_M[n\alpha, \sigma, \bar{c}_B(\alpha, \sigma)]}{\partial \bar{c}} \frac{\partial \bar{c}_B}{\partial \alpha} \\
&= u_c(c_B, l_B, \bar{c}_B) \cdot \left[ n l_B + \frac{u_{\bar{c}}(c_B, l_B, \bar{c}_B)}{u_c(c_B, l_B, \bar{c}_B)} \frac{\partial \bar{c}_B}{\partial \alpha} \right] \\
&= u_c(c_B, l_B, \bar{c}_B) \cdot \left[ n l_B - \delta_B \frac{\partial \bar{c}_B}{\partial \alpha} \right]
\end{aligned} \tag{A.3}$$

where  $\delta_B = -u_{\bar{c}}(c_B, l_B, \bar{c}_B)/u_c(c_B, l_B, \bar{c}_B)$ .

Given that  $\partial \bar{c}_B / \partial \alpha > 0$  we certainly have from (A.3)  $\partial v_B / \partial \alpha < 0$  for  $n \leq \underline{n}_B$  (because  $l_B = 0$ ); whilst for  $n > \underline{n}_B$  but  $n \approx \underline{n}_B$  we have  $l_B \approx 0$  and so it is also the case that  $\partial v_B / \partial \alpha \approx -u_c \delta_B \partial \bar{c}_B / \partial \alpha < 0$ . ■

**Derivation of the Adjusted Roy's identity** Differentiating the equilibrium indirect utility functions w.r.t. the basic income yields

$$\begin{aligned}
\frac{\partial v_N(n\alpha, \sigma)}{\partial \sigma} &= \frac{\partial v_M(n\alpha, \sigma, \bar{c}_N)}{\partial \sigma} + \frac{\partial v_M(n\alpha, \sigma, \bar{c}_N)}{\partial \bar{c}} \frac{\partial \bar{c}_N}{\partial \sigma} \\
&= u_c(\bar{c}_N, l_N, \bar{c}_N) \cdot \left[ 1 + \frac{u_{\bar{c}}(\bar{c}_N, l_N, \bar{c}_N)}{u_c(\bar{c}_N, l_N, \bar{c}_N)} \frac{\partial \bar{c}_N}{\partial \sigma} \right]
\end{aligned} \tag{A.4}$$



$$\begin{aligned}\frac{\partial v_B(n, \alpha, \sigma)}{\partial \sigma} &= \frac{\partial v_M(n\alpha, \sigma, \bar{c}_B)}{\partial \sigma} + \frac{\partial v_M(n\alpha, \sigma, \bar{c}_B)}{\partial \bar{c}} \frac{\partial \bar{c}_N}{\partial \sigma} \\ &= u_c(\bar{c}_B, l_B, \bar{c}_B) \cdot \left[ 1 + \frac{u_{\bar{c}}(\bar{c}_B, l_B, \bar{c}_B)}{u_c(\bar{c}_B, l_B, \bar{c}_B)} \frac{\partial \bar{c}_B}{\partial \sigma} \right]\end{aligned}\quad (\text{A.5})$$

Combining these expressions with those in Result 1 we can derive the distortion-adjusted Roy's identity.

- *Narrow Case.* From (A.1) and (A.4)

$$\frac{\partial v_N}{\partial(n\alpha)} \left( \frac{\partial v_N}{\partial \sigma} \right)^{-1} = \frac{l_N - \delta_N \frac{\partial \bar{c}_N}{\partial(n\alpha)}}{1 - \delta_N \frac{\partial \bar{c}_N}{\partial \sigma}} = l_N - \left[ l_N - \frac{l_N - \delta_N \frac{\partial \bar{c}_N}{\partial(n\alpha)}}{1 - \delta_N \frac{\partial \bar{c}_N}{\partial \sigma}} \right] = l_N - \frac{\delta_N \left[ \frac{\partial \bar{c}_N}{\partial(n\alpha)} - l_N \frac{\partial \bar{c}_N}{\partial \sigma} \right]}{1 - \delta_N \frac{\partial \bar{c}_N}{\partial \sigma}} \quad (\text{A.6})$$

- *Broad Case.* From (A.3) and (A.5)

$$\frac{\partial v_B}{\partial \alpha} \left( \frac{\partial v_B}{\partial \sigma} \right)^{-1} = \frac{nl_B - \delta_B \frac{\partial \bar{c}_B}{\partial \alpha}}{1 - \delta_B \frac{\partial \bar{c}_B}{\partial \sigma}} = nl_B - \left[ nl_B - \frac{nl_B - \delta_B \frac{\partial \bar{c}_B}{\partial \alpha}}{1 - \delta_B \frac{\partial \bar{c}_B}{\partial \sigma}} \right] = nl_B - \frac{\delta_B \left[ \frac{\partial \bar{c}_B}{\partial \alpha} - nl_B \frac{\partial \bar{c}_B}{\partial \sigma} \right]}{1 - \delta_B \frac{\partial \bar{c}_B}{\partial \sigma}} \quad (\text{A.7})$$

### Derivation of Proposition 3.

- *Proposition 3(a): Narrow Social Optimum.* The first-order condition resulting from  $l_{SN}(n\alpha, \sigma) := \text{Arg max}_l u(n\alpha l + \sigma, l, n\alpha l + \sigma)$  is

$$(u_c + u_{\bar{c}}) n\alpha + u_l \leq 0 \Rightarrow n\alpha \left( 1 + \frac{u_{\bar{c}}}{u_c} \right) \leq -\frac{u_l}{u_c} ; l_{SN} \geq 0 \quad (\text{A.8})$$

where the pair of inequalities hold with complementary slackness. Suppose we impose a (productivity dependent) Pigovian tax on earnings, where the net-of-Pigovian tax rate is  $p_n \in (0, 1)$ . One can readily show that the Nash equilibrium is now characterised by

$$n(\alpha p_n) \leq -\frac{u_l(n\alpha p_n l_N + \sigma_n, l_N, n\alpha p_n l_N + \sigma_n)}{u_c(n\alpha p_n l_N + \sigma_n, l_N, n\alpha p_n l_N + \sigma_n)} ; l_N \geq 0 \quad (\text{A.9})$$

Substituting in the budget-balancing condition  $\sigma_n = \sigma + (1 - p_n)n\alpha l_N$  then yields

$$n\alpha p_n \leq -\frac{u_l(n\alpha l_N + \sigma, l_N, n\alpha l_N + \sigma)}{u_c(n\alpha l_N + \sigma, l_N, n\alpha l_N + \sigma)} ; l_N \geq 0 \quad (\text{A.10})$$

Given that indifference curves are strictly convex it must hold that setting

$$1 - p_n = - \frac{u_{\bar{c}}(n\alpha l_{SN} + \sigma, l_{SN}, n\alpha l_{SN} + \sigma)}{u_c(n\alpha l_{SN} + \sigma, l_{SN}, n\alpha l_{SN} + \sigma)} \quad (\text{A.11})$$

yields  $l_N = l_{SN}$ . ■

- *Proposition 3(b): Broad Social Optimum.* Letting  $c_n(l) = n\alpha l + \sigma$ , the first-order condition resulting from  $l_{SB} := \text{Arg max}_l u(c_n, l, \bar{c})$  s.t.  $dc_n = d\bar{c}$  is

$$\begin{aligned} \left( u_c + u_{\bar{c}} \frac{d\bar{c}}{dc_n} \right) \frac{dc_n}{dl} + u_l &\leq 0 \\ \Rightarrow (u_c + u_{\bar{c}})n\alpha + u_l &\leq 0 \\ \Rightarrow n\alpha \left( 1 + \frac{u_{\bar{c}}}{u_c} \right) &\leq \frac{u_l}{u_c} ; l_{SB} \geq 0 \end{aligned} \quad (\text{A.12})$$

where the pair of inequalities hold with complementary slackness.

#### Derivation of Proposition 4 (Optimal Tax Expression)

- *Narrow Case.*

$$\begin{aligned} 0 &= \int_0^\infty G'(v_N) \left\{ \frac{\partial v_N}{\partial(n\alpha)} n - \frac{\partial v_N}{\partial \sigma} \frac{d\sigma_N}{d(1-\alpha)} \right\} dF(n) \\ &= \int_0^\infty G'(v_N) \frac{\partial v_N}{\partial \sigma} \left\{ \left( z_N - \tilde{Z}_N \right) - \Delta_N + (1 - \hat{\alpha}_N) \frac{d\tilde{Z}_N}{d\alpha} \right\} dF(n) \end{aligned} \quad (\text{A.13})$$

To transition from the first to second equality we substituted in (i)  $n\partial v_N/\partial(n\alpha) = (\partial v_N/\partial \sigma) \cdot (nl_N - \Delta_N)$ ; and (ii)  $d\sigma_N/d(1-\alpha) = \tilde{Z}_N - (1-\alpha)d\tilde{Z}_N/d\alpha$ . Simple manipulation of (A.13) then yields

$$(1 - \hat{\alpha}_N) = \int_0^\infty \left\{ \underbrace{\left[ \frac{G'(v_N) \frac{\partial v_N}{\partial \sigma}}{\int_0^\infty G'(v_N) \frac{\partial v_N}{\partial \sigma} dF(n)} \right]}_{g_N(n)} \cdot \left[ \left( 1 - \frac{z_N}{\tilde{Z}_N} \right) + \left( \frac{\Delta_N}{\tilde{Z}_N} \right) \right] \right\} dF(n) \cdot \left( \frac{1}{\tilde{Z}_N} \frac{d\tilde{Z}_N}{d\alpha} \right)^{-1}$$

and so

$$\frac{1 - \hat{\alpha}_N}{\alpha_N} = \int_0^\infty \left[ 1 - g_N \left( \frac{z_N}{\tilde{Z}_N} \right) + g_N \left( \frac{\Delta_N}{\tilde{Z}_N} \right) \right] dF(n) \underbrace{\left( \frac{\alpha}{\tilde{Z}_N} \frac{d\tilde{Z}_N}{d\alpha} \right)^{-1}}_{1/e_N} \quad \blacksquare \quad (\text{A.14})$$

- *Broad Case.*

$$\begin{aligned}
0 &= \int_0^\infty G'(v_B) \left\{ \frac{\partial v_B}{\partial \alpha} - \frac{\partial v_B}{\partial \sigma} \frac{d\sigma_B}{d(1-\alpha)} \right\} dF(n) \\
&= \int_0^\infty G'(v_B) \left\{ \left( z_B - \tilde{Z}_B \right) - \Delta_B + (1 - \hat{\alpha}_B) \frac{d\tilde{Z}_B}{d\alpha} \right\} dF(n)
\end{aligned} \tag{A.15}$$

To transition from the first to second equality we substituted in (i)  $\partial v_B / \partial \alpha = n l_B - \Delta_B$ ; and (ii)  $d\sigma_B / d(1 - \alpha) = \tilde{Z}_B - (1 - \alpha) d\tilde{Z}_B / d\alpha$ . Simple manipulation of (A.15) then yields

$$(1 - \hat{\alpha}_B) = \int_0^\infty \left\{ \underbrace{\left[ \frac{G'(v_B) \frac{\partial v_B}{\partial \sigma}}{\int_0^\infty G'(v_B) \frac{\partial v_B}{\partial \sigma} dF(n)} \right]}_{g_B(n)} \left[ \left( 1 - \frac{z_B}{\tilde{Z}_B} \right) + \Delta_B \right] \right\} dF(n) \left( \frac{1}{\tilde{Z}_B} \frac{d\tilde{Z}_B}{d\alpha} \right)^{-1}$$

and so

$$\frac{1 - \hat{\alpha}_B}{\hat{\alpha}_B} = \int_0^\infty \left[ 1 - g_B \left( \frac{z_B}{\tilde{Z}_B} \right) + g_B \Delta_B \right] dF(n) \underbrace{\left( \frac{\hat{\alpha}_B}{\tilde{Z}_B} \frac{d\tilde{Z}_B}{d\alpha} \right)^{-1}}_{1/e_B} \blacksquare \tag{A.16}$$